## 臺灣綜合大學系統 105 學年度學士班轉學生聯合招生考試試題

| 科目名稱 | 微積分 B | 類組代碼 | E00 |
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| | | 科目碼 | E0012 |
| ※本項考試依簡章規定各考科均「不可以」使用計算機 | | 本試題共計 **1** 頁 | |

※ 注意：請於答案卷作答並提供完整計算過程，否則不予計分。

1. Let $f(x) = x^2 \left[ 1 - \cos\left(\dfrac{3}{x}\right) \right]$, $x \neq 0$.

   1-1.(5%) Evaluate the limit. $\lim\limits_{x \to 0} f(x)$

   1-2.(5%) Evaluate the limit. $\lim\limits_{x \to \infty} f(x)$

   1-3.(10%) Evaluate the limit. $\lim\limits_{x \to 0} f'(x)$

2. Let $f(x) = \exp\left(1 - \dfrac{x^2}{8}\right)$.

   2-1.(5%) Find the intervals of increases or decreases of $f(x)$.

   2-2.(5%) Find the intervals of concave up or concave down of $f(x)$.

   2-3.(10%) Sketch the graph of $f(x)$, $x \in [-5, 5]$. Label the points of local maximum

   or minimum, points of inflection and asymptotes on the graph. ($e \approx 2.71$, $\sqrt{e} \approx 1.65$.)

3. Let $f(x, y) = 8x^3 - 12xy + y^3$.

   3-1.(5%) Find the tangent plane of the surface $z = f(x, y)$ at $(1, 4)$.

   3-2.(5%) In what direction does $f(x, y)$ have minimum rate of change at $(1, 4)$?

   3-3.(10%) Find the local maximum, local minimum and saddle points of $f(x, y)$.

4.(10%) Evaluate the indefinite integral. $\displaystyle\int \dfrac{(\ln x)^2}{x^2} dx$

5.(10%) Use third order Taylor polynomial to approximate the number $\sqrt{4.05}$.

   (Put the answer in the form $\sqrt{4.05} \approx 2 \pm \dfrac{1}{\boxed{\phantom{x}}} \pm \dfrac{1}{\boxed{\phantom{xx}}} \pm \dfrac{1}{\boxed{\phantom{xx}}}$.)

6.(10%) The Cobb-Douglas production function is $P = x^{\frac{1}{4}} y^{\frac{1}{2}} z^{\frac{1}{4}}$ subject to total budget

   constraint $3x + 4y + 5z = 240$. ($x$: capital, $y$: labor, $z$: resource.)

   Use Lagrange multiplier to find values of $x, y, z$ such that $P$ is maximized.

7.(10%) Evaluate the double integral. $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \dfrac{1}{\sqrt{x}(1 + y^4)} dy\,dx$