

臺灣綜合大學系統 106 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	
		科目碼	E0011

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 2 頁

1. (10 points) Given a curve  $C$  in  $\mathbb{R}^2$  defined by

$$\ln(1 - x^3 + y^3) - 4 = 0.$$

Find the point on  $C$  at which the tangent line is vertical.

2. (10 points) If the function

$$f(x) = \begin{cases} \frac{\sin(4x)+a-2b}{3x} & x \neq 0 \\ 2a + b & x = 0 \end{cases}$$

is continuous at  $x = 0$ , then  $(a, b) = ?$

3. (10 points) Write down the first three terms (three lowest order terms) of the Taylor series of  $\frac{\tan^{-1}(2x)}{1-x}$  at 0. (Hint:  $\tan^{-1} u = \int ? du$ )

4. (10 points) Evaluate the following integral:

$$\int_0^1 \int_{\sqrt{x}}^1 x \cos(y^5 + 2) dy dx.$$

5. (10 points) From the equation

$$e^{x^2} + y^2 \sin(2x) = 4y,$$

Solve  $\frac{dy}{dx}$  in terms of  $x$  and  $y$

6. (10 points) Find all values of  $a$  so that the series

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{3a-1} + 3}\right)$$

is divergent.

背面有題，請繼續作答。

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7. (10 points) Compute the following improper integral

$$\int_0^{\infty} e^{-4x^2} dx.$$

8. (10 points) Compute the line integral

$$\oint_C \vec{F} \cdot d\vec{r},$$

where

$$\vec{F}(x, y) = (4y + 6ye^{2x}, 6x + 3e^{2x})$$

and  $C$  is the closed loop formed by traveling from  $(-2, 0)$  to  $(4, 0)$  to  $(3, 3)$  to  $(-1, 3)$  and back to  $(-2, 0)$  by straight lines.

9. (10 points) Given the function  $F : \mathbb{R}^3 \rightarrow \mathbb{R}$  by

$$F(x, y, z) = e^{x+y^2+\cos z}.$$

At  $(0, 0, 0)$ , find the direction along which the function *decreases* most rapidly and find the corresponding rate of change.

10. (10 points) A rectangular box is formed by cutting four equal corners from a square of side 3 and then folding up (see the figure below). Find the maximum possible volume of the box.

