臺灣綜合大學系統106學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	A07/D01
		科目碼	A0702

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 1 頁

In the following, $\mathbb{R}^{m\times n}$ denotes the class of all $m \times n$ real matrices. It is a real vector space with respect to matrix addition and scalar multiplication. Vectors in $\mathbb{R}^n$ will be regarded as column vectors.

1. (a) Let $A \in \mathbb{R}^{n\times n}$. Show that if A is symmetric and satisfies $Ax \cdot x = 0$ for all $x \in \mathbb{R}^n$, then $A = 0$. (10 points)
(b) Give an example to show that the assumption "symmetric" in (a) is necessary. (10 points)

2. Let X be the linear subspace of $\mathbb{R}^{2\times 3}$ containing all matrices whose columns add to $0 \in \mathbb{R}^2$. For example, $\begin{bmatrix} 1 & 2 & -3 \\ -2 & -2 & 4 \end{bmatrix} \in X$ since $\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Similarly let Y be the subspace of $\mathbb{R}^{2\times 3}$ containing all matrices whose rows add to $0 \in \mathbb{R}^3$. Answer the following questions with reasons.
(a) What is the dimension of X. (10 points)
(b) What is the dimension of $X + Y$. (10 points)

3. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. For $u, v \in \mathbb{R}^3$, define $\langle u, v\rangle = u^T Av$, then $\langle \cdot, \cdot \rangle$ is an inner product on $\mathbb{R}^3$ (no need to prove it). Under this inner product, find the orthonormal basis $\{f_1, f_2, f_3\}$ by applying the Gram-Schmidt process to $\{(1,0,0)^T, (0,1,0)^T, (0,0,1)^T\}$. (15 points)

4. In $\mathbb{R}^2$, let L be the line $y = mx$, where $m \in \mathbb{R}$. Find the matrix $A \in \mathbb{R}^{2\times 2}$ so that $x \mapsto Ax$ is the reflection on $\mathbb{R}^2$ about L. (15 points)

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$. Under the ordered basis $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ of $\mathbb{R}^{2\times 2}$, compute the 4×4 matrix representing the linear transformation $M \mapsto AM$ (left multiplication by A) on $\mathbb{R}^{2\times 2}$. (15 points)

6. Find an $A \in \mathbb{R}^{2\times 2}$ (other than $A = I$) such that $A^3 = I$. (15 points)