

臺灣綜合大學系統 106 學年度學士班轉學生聯合招生考試試題

科目名稱	基礎數學	類組代碼	D25
		科目碼	D2591

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 1 頁

Instruction: SHOW YOUR WORK.

Partial credit may be awarded if a substantial part of the answer is provided.

1. (10pts) Prove, by the ε - δ argument, that $\lim_{x \rightarrow 4} x^2 = 16$.

2. (10ts) Let

$$f(x) = \begin{cases} x^3 \sin(1/x), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Discuss the differentiability of $f'(x)$ at $x = 0$, i.e., the existence of $f''(0)$.

3. (10pts) Give a sequence of continuous functions for which $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$, provided that $\lim_{n \rightarrow \infty} f_n(x)$ is continuous.

4. (10pts) Evaluate $\int_0^{\infty} e^{-x^2} dx$.

5. (7+3pts) Find (i) the Maclaurin series for $\tan^{-1} x$ and (ii) its radius of convergence.

6. (10pts) Calculate the curvature κ at each point of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

7. (5+5pts) Let

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

(i) Find $f_x(x, y)$ and $f_y(x, y)$. (ii) Discuss the differentiability of $f(x, y)$ at $(0, 0)$.

8. (10pts) A rectangular box, open at the top, is to be made from 12ft² of cardboard. Find the maximum volume of such a box by the second derivatives test.

9. (10pts) Evaluate $\int_0^1 \int_y^1 e^{y/x} dx dy$.

10. (10pts) Evaluate $\oint_C 2y^2 dx + 3xy dy$, where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.