1. (20%)(a) Solve the initial-value problem

   $\frac{d^2x}{dt^2} + \omega^2 x = F_0 \cos^2 \gamma t$, $x(0) = 0$, $x'(0) = 0$.　　　(10%)

   (b) Discuss under what frequency of γ the system is in pure resonance.　　(10%)

2. (20%) Find a matrix **A** such that

$$\mathbf{A} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x - y \\ x + y + 2w \end{bmatrix}$$

3. (20%) Let **u** be an $n \times 1$ column matrix satisfying $\mathbf{u}^T\mathbf{u}=1$. The $n \times n$ matrix $\mathbf{H}=\mathbf{I}_n - 2\mathbf{u}\mathbf{u}^T$ is called a Householder matrix. Prove that **H** is symmetric and nonsingular ($\mathbf{I}_n$ is an $n \times n$ identity matrix)

4. (20%) Solve the diffusion problem $u_t = ku_{xx}$ in $0 < x < L$, with the mixed boundary conditions $u(0,t) = u_x(L,t) = 0$ and initial condition $u(x,0) = f(x)$

5. (20%) Let $f(x,y) = \ln(x^2 + y^2 + 1) + e^{2xy}$

   (a) Find the gradient of f at the point $(0, -2)$　　(7%)

   (b) Find the directional derivative of f at the point $(0, -2)$ in the direction of the vector $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$　　(7%)

   (c) Find the maximum value of the directional derivative at the point $(0, -2)$.　(6%)