

臺灣綜合大學系統 107 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013
※本項考試依簡章規定各考科均「不可以」使用計算機		本科試題共計 1 頁	

題號標示清楚，寫出計算過程否則不予計分，答案儘可能化簡。

1.(10%) Differentiate the function. $f(x) = x^9 + 5^x + \log_2 x + \log_x 7$

2.(10%) Find the maximum value of the function. $f(x) = \frac{\ln x}{\sqrt[3]{x}}$

3.(10%) Evaluate the limit. $\lim_{x \rightarrow 0} \frac{\sin(2 \tan^{-1} x) - \tan(\sin^{-1} 2x)}{x^3}$

4.(10%) Evaluate the limit. $\lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{(n+1)(n+3)(n+5) \cdots (3n-1)}$

5.(10%) Evaluate the integral. $\int \frac{dx}{(x^2+2)^{\frac{5}{2}}} dx$

Hint: $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$, $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

6.(10%) Use $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ to evaluate the integral $\int_0^1 \frac{\ln(1+x)}{x} dx$.

7.(10%) Find the area of the region bounded by the polar curve $r = \frac{3}{2 + \sin \theta}$.

Hint: sketch the curve first.

8.(10%) Let $F(x, y, z) = xy + z^3 + xyz - 5$. Suppose a function $f(x, y)$ is defined in a neighborhood of $(2, 1)$ such that $F(x, y, f(x, y)) = 0$. Estimate $f(1.97, 1.04)$ by linear approximation.

9.(10%) Evaluate the double integral $\iint_R \frac{xy}{y^4 - x^4} dA$, where R is the region in the first quadrant bounded by $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, $y^2 - x^2 = 1$, $y^2 - x^2 = 4$.

10.(10%) Evaluate the line integral $\int_C \frac{(x-y)dx + (x+y)dy}{x^2 + y^2}$, where C is the curve $\mathbf{r}(t) = \langle 2-t, 1+2\sqrt{t} \rangle$, $0 \leq t \leq 1$.