

臺灣綜合大學系統 107 學年度學士班轉學生聯合招生考試試題

科目名稱	基礎數學	類組代碼	D25
		科目碼	D2591

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 2 頁

一、選擇題 (每題 5 分, 共 25 分。)

每題的正確選項可能不只一個, 答案完全正確得 5 分, 否則得零分!

1. Which of the following must be true? ( $\mathbb{R}$  denotes the set of real numbers.)

A.  $\int a^x dx = \frac{a^{x+1}}{x+1} + c$ , where  $0 < a \neq 1$ ,  $c \in \mathbb{R}$  and  $x \in \mathbb{R}$ .

B.  $\int a^x dx = a^x + c$ , where  $0 < a \neq 1$ ,  $c \in \mathbb{R}$  and  $x \in \mathbb{R}$ .

C.  $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c$ , where  $\alpha, c \in \mathbb{R}$  and  $x \in (0, \infty)$ .

D.  $\int \frac{1}{x} dx = \ln|x| + c$ , where  $c \in \mathbb{R}$  and  $x \neq 0$ .

2. Let  $f$  be a function defined on  $\mathbb{R}$ . Which statements of the following are equivalent to

$$\lim_{x \rightarrow 1} f(x) = 2?$$

A.  $\exists \varepsilon > 0, \forall \delta > 0$  such that  $|x - 1| < \delta$  implies  $|f(x) - 2| < \varepsilon$ .

B.  $\exists \varepsilon > 0, \forall \delta > 0$  such that  $0 < |x - 1| < \delta$  implies  $|f(x) - 2| < \varepsilon$ .

C.  $\forall \varepsilon > 0, \exists \delta > 0$  such that  $|x - 1| < \delta$  implies  $|f(x) - 2| < \varepsilon$ .

D.  $\forall \varepsilon > 0, \exists \delta > 0$  such that  $0 < |x - 1| < \delta$  implies  $|f(x) - 2| < \varepsilon$ .

3. Let  $\{a_n\}$  be a sequence of real numbers. Which of the following must be true?

A. If  $\lim_{n \rightarrow \infty} a_n \neq 0$  then  $\sum_{n=1}^{\infty} a_n$  diverges.

B. If  $a_n < 0, \forall n \in \mathbb{N}$  and  $\lim_{n \rightarrow \infty} a_n = a$ , then  $a < 0$ .

C. If  $\{a_n\}$  is an increasing sequence, then  $\lim_{n \rightarrow \infty} a_n = +\infty$ .

D. If  $\lim_{n \rightarrow \infty} |a_n|$  exists, then  $\lim_{n \rightarrow \infty} a_n$  exists.

4. Let  $f$  be a function defined on  $[0, \infty)$ . Which of the following must be true?

A. If  $f$  is Riemann integrable on  $[0, 1]$ , then  $|f|$  is Riemann integrable on  $[0, 1]$ .

B. If  $|f|$  is Riemann integrable on  $[0, 1]$ , then  $f$  is Riemann integrable on  $[0, 1]$ .

C. If  $f$  is continuous on  $[0, \infty)$ , and the improper Riemann integral of  $f$  on  $[0, \infty)$  exists, then the improper Riemann integral of  $|f|$  on  $[0, \infty)$  exists

D. If  $f$  is continuous on  $[0, \infty)$ , and the improper Riemann integral of  $|f|$  on  $[0, \infty)$  exists then the improper Riemann integral of  $f$  on  $[0, \infty)$  exists.

背面有題, 請繼續作答。

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5. Let  $f$  be a function defined on  $\mathbb{R}^2$ . Which of the following must be true?
- A. If  $f$  is continuous at  $(0, 0)$ , then the directional derivative of  $f$  at  $(0, 0)$  exists in all directions.
- B. If the directional derivative of  $f$  at  $(0, 0)$  exists in all directions, then  $f$  is continuous at  $(0, 0)$ .
- C. If the directional derivative of  $f$  at  $(0, 0)$  exists in all directions, then  $f$  is differentiable at  $(0, 0)$ .
- D. None of the above.

二、填充題：(每個空格 5 分，共 30 分)

1. Let  $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ ,  $\forall x \in \left[\frac{1}{2}, 2\right] \equiv I$  and  $S \equiv \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq f(x) \text{ and } x \in I\}$ .

(a). The volume of the solid generated by revolving  $S$  about  $y$ -axis = ?.

(b). The arc length of the graph of  $f$  on  $I$  = ?.

2. Suppose that  $\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} dy dx = \int \int_S \frac{x e^{2y}}{4-y} dA = \int_0^4 \int_0^{f(y)} \frac{x e^{2y}}{4-y} dx dy = B$ .

Then the region  $S = \underline{\quad}$ ,  $f(y) = \underline{\quad}$  and  $B = \underline{\quad}$ .

3.  $\int \int \int_{\Omega} x^4 y^2 z^3 dV = \underline{\quad}$ , where  $\Omega = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 3\}$ .

三、計算題：(45 分。僅有答案，沒有計算過程得零分。)

1. Consider the power series  $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$ .

(a). Evaluate the radius of convergence. (5 分)

(b). Find the set of convergence. (10 分)

2. Consider the function  $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

(a). Evaluate  $\lim_{x \rightarrow 0} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$ . (5 分)

(b). Find  $f'(x)$  for all  $x \in \mathbb{R}$ . (10 分)

(c). Draw the graph of  $f$  on  $\mathbb{R}$ . (15 分)