臺灣綜合大學系統108學年度學士班轉學生聯合招生考試試題

科目名稱	基礎數學	類組代碼	D25
		科目碼	D2591

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 1 頁

(15%) 1. Evaluate the following limits:

(a)(5%) $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$.

(b)(10%) $\lim_{x\to(\pi/2)^-}(\tan x)^{\cos x}$.

(15%) 2. Test the following series for convergence or divergence:

(a)(5%) $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$.

(b)(10%) $\sum_{n=1}^{\infty}\frac{n^n}{n!}$.

(15%) 3. Evaluate the following integrals:

(a)(5%) $\int_0^1(x^2+1)e^{-x}\,dx$.

(b)(10%) $\int_2^3\frac{x+5}{x^2+x-2}\,dx$.

(15%) 4. If $g(s,t)=f(s^2-t^2,t^2-s^2)$ and suppose g and f are differentiable. Show that g satisfies the equation

$$t\frac{\partial g}{\partial s}+s\frac{\partial g}{\partial t}=0.$$

(15%) 5. Let the sequence $\{a_n\}$ be

$$a_1=2 \text{ and } a_n=\frac{1}{3-a_{n-1}}, \forall n\geq 2$$

(a)(10%) Use the Monotonic Sequence Theorem to show that $\{a_n\}$ is convergent.

(b)(5%) Find $\lim_{n\to\infty}a_n$.

(15%) 6. (a)(5%) Prove that if f is a continuous function, then

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx.$$

(b)(10%) Use part(a) to show that

$$\int_0^{\pi/2}\frac{\sin^n x}{\sin^n x+\cos^n x}\,dx=\frac{\pi}{4}, \forall n>0.$$

(10%) 7. Suppose f and g are differentiable functions with $f(g(x))=x$ and $f'(x)=1+[f(x)]^2$. Show that $g'(x)=1/(1+x^2)$.