臺灣綜合大學系統108學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	D25
		科目碼	D2592
※ 本項考試依簡章規定各考科均「不可以」使用計算機		本科試題共計 3 頁	

一、單選題 (30%，每題恰有一個選項是正確的，請依序標明題號後在「答案卷」上作答。)

(1). (5 pts) Let $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | x_1 - x_2 + x_3 = 0\}$, denote $S^\perp$ be the set of all vectors in $\mathbb{R}^3$ that are orthogonal to every vector in S. Which of the following is correct?

(A) $(1,1,0) \in S^\perp$.

(B) $(1,-1,1) \in S^\perp$.

(C) S is a subspace of $\mathbb{R}^3$ and $\dim S = 1$.

(D) Let $(1,1,0) = \mathbf{w} + \mathbf{z}$ such that $\mathbf{w} \in S$ and $\mathbf{z} \in S^\perp$, then $\mathbf{z} = \left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right)$.

(E) $\text{Span}\{(-1,0,1), (-1,1,-1)\} \subset S$.

(2). (5 pts) Let $A \in M^{4\times4}(\mathbb{R})$ and $\det A = -4$. Which of the following statements is correct?

(A) The adjoint matrix $\text{adj}\, A$ has determinant -64.

(B) $\det A^T = 16$.

(C) $\det(-A) = -4$.

(D) If $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{bmatrix}$ and $\mathbf{b} \in \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$. Then $\det A = \det \begin{bmatrix} \mathbf{a}_1 + \mathbf{b} & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{bmatrix}$.

(E) A is singular.

(3). (5 pts) Suppose the matrix $\begin{bmatrix} 1 & 2 & 3 & 1 & b \\ 2 & 5 & 3 & a & 0 \\ 1 & 0 & 8 & 6 & c \end{bmatrix}$ can be transformed to the reduced row echelon form $\begin{bmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & d & -1 \\ 0 & 0 & 1 & 1 & e \end{bmatrix}$. Which of the following is correct?

(A) $a = 1$. (B) $b = 3$. (C) $c = 20$. (D) $d = -1$. (E) $e = 2$.

(4). (5 pts) Let $A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and $E = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$. Which of the following statements is correct?

(A) A is similar to D.

(B) B is similar to D.

(C) A is similar to E.

(D) C is similar to D.

(E) E is similar to D.

(5). (5 pts) Which of the following statements is correct?

(A) If the reduced row echelon form of $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ contains a zero,row, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(B) If V and W are subspaces of $\mathbb{R}^n$ having the same dimension, then $V = W$.

(C) Let $M^{m\times n}$ denote the space of all $m \times n$ matrices. A matrix representation of a linear operator on $M^{m\times n}$ is an $m \times n$ matrix.

(D) A mapping $f : \mathbb{R}^n \to \mathbb{R}^m$ is uniquely determined by its images of the standard vectors in $\mathbb{R}^n$.

(E) Let A, B, and C be any matrices such that the product ABC is defined. Then $\text{rank}(ABC) \leq \text{rank}\, B$.

(6). (5 pts) Which of the following statement is wrong?

(A) For any $n \times n$ matrix A, the columns of A are linearly independent if and only if the rows of A are linearly independent.

(B) For an $m \times n$ matrix A, the nullity of A equals the nullity of its transpose A^T.

(C) An $m \times n$ matrix A defines some linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$. T_A is onto if and only if $\text{rank}\, A = m$.

(D) A set S of vectors forms a basis for a subspace V of $\mathbb{R}^n$ if and only if the vectors of S are linearly independent and the number of vectors in S equals the dimension of V.

(E) The set $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | 3x_1 + 2x_2 - x_3 = 1\}$ is not a subspace of $\mathbb{R}^3$.

二、填充題 (70%，請依序標明格號後在「答案卷」上作答。)

- (10 pts) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Let $A = QR$ be the QR-decomposition of A. Then $Q =$ ____(7)____, and $R =$ ____(8)____.

- (15 pts) Let $A = \begin{bmatrix} 2 & -2 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix}$. Let n be the number of the Jordan blocks of A. Then $n =$ ____(9)____.

背面有題，請繼續作答。

- (15 pts) Let $M = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$. Let $M = U\Sigma V^T$ be the singular value decomposition of M. Then $U =$ ___**(10)**___, $\Sigma =$ ___**(11)**___, and $V =$ ___**(12)**___.

- (10 pts) Let $A = \begin{bmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$. Then $\det A =$ ___**(13)**___.

- Suppose $V = M^{2\times 2}(\mathbb{R})$. Let V_1, V_2 be two subspaces of V defined by

$$V_1 = \left\{ \begin{bmatrix} a+b & 2a+3b \\ b & b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\},$$

$$V_2 = \left\{ \begin{bmatrix} 0 & a \\ -a+2b & b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}.$$

Then

(i) (8 pts) $\dim V_1 + \dim V_2 =$ ___**(14)**___.

(ii) (8 pts) $\dim(V_1 + V_2) =$ ___**(15)**___.

(iii) (4 pts) $\dim(V_1 \cap V_2) =$ ___**(16)**___.