

臺灣綜合大學系統109學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 1 頁

Answer without complete work shown receives no credit. No electronic devices allowed.

1. (10 points) Find the following limits:

(a) (5 points) $\lim_{x \rightarrow 1} \frac{e^x - 1}{x}$

(b) (5 points) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x)$

2. (10 points) Find the tangent line to the curve defined by the equation $4^{x/y} = x - y$ when $x = 0$.

3. (10 points) Compute the definite integral

$$\int_0^{4\sqrt{2}} \int_{x^{2/5}}^2 \frac{e^y}{\sqrt{y}} dy dx.$$

4. (10 points) Find the arc length of the polar curve $r = \theta$ with $0 \leq \theta \leq \pi$.

5. (10 points) Find the volume of the solid of revolution about the x -axis bounded by the curve $f(x) = \frac{1}{x\sqrt{x^2 + 1}}$ with $x \in [1, \infty)$.

6. (10 points) Consider the power series $\sum_{n=1}^{\infty} \frac{(-3)^n(x+2)^n}{\sqrt{n+1}}$. Find its maximal domain of convergence.

7. (10 points) Recall that the hyperbolic tangent function

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \quad x \in \mathbb{R}$$

is one to one and has the inverse function $f(x) = \tanh^{-1} x$. Find the Maclaurin series of $f(x) = \tanh^{-1} x$.

8. (10 points) Find the absolute maximum of $f(x, y) = e^{-xy}$ on the region $x^2 + 4y^2 \leq 1$.

9. (10 points) Evaluate the line integral $\int_C y^3 dx - x^3 dy$, where C is the circle $x^2 + y^2 = 8$ with positive orientation.

10. (10 points) Let $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} - 2z\mathbf{k}$. Suppose that the closed surface S is defined by the paraboloid $y = x^2 + z^2$ with $0 \leq y \leq 1$. Find the flux of \mathbf{F} outward across S .