臺灣綜合大學系統109學年度學士班轉學生聯合招生考試試題

科目名稱	微積分B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 2 頁

Answer without complete work shown receives no credit. 所有計算過程都必須詳細列出，否則不予計分。

1. (10 points) Find the following limits.

   (a)
   $$\lim_{x\to 2}\frac{x+4}{x-7}.$$

   (b)
   $$\lim_{x\to 0}(1-7x)^{\frac{4}{x}}.$$

2. (10 points) Evaluate the following integrals

   (a)
   $$\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx.$$

   (b)
   $$\int_4^5 \frac{x-3}{\sqrt{x^2-6x+13}}\,dx.$$

3. (10 points) Evaluate $\frac{\partial u}{\partial x}|_{(0,1)}$ for
   $$u(x,y)=\left(e^x-\frac{y}{6}\right)\int_{-2}^{2x}\sqrt{4-t^2}\,dt.$$

   Note: Answer must in numerical expression and natural constants like π, e, ... etc.

4. (10 points) Evaluate $\frac{\partial u}{\partial y}|_{(2,0)}$ for
   $$u(x,y)=h(x^2+y^2,3x-4y)$$

   where
   $$h(s,t)=\frac{t}{4s}-t^2+\frac{\tan s}{\ln s}.$$

5. (10 points) Find the volume of the solid generated by rotating the curve
   $$y=\sin(x^2)$$

   over $0\le x\le 1$ around the $y-$axis.

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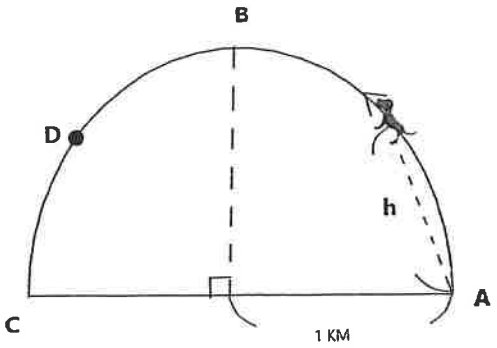
6. (10 points) Evaluate the infinite sum

$$\sum_{k=0}^{\infty} \frac{1}{(k+2)k!}$$

by some manipulations of the Taylor series of $f(x) = xe^x$.

7. (10 points) A dog is running along a semi-circular track with radius 1 km in counterclockwise direction with speed 0.1 km per minute (See Figure).



Let h be the distance between the dog and point A. Find the rate of change of h at point D, half way between B and C.

8. (10 points) Find $(f^{-1})'(5)$ for

$$f(x) = x^5 + 2x^3 + 2x.$$

9. (10 points) Evaluate

$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{9-x^2}} e^{x^2+y^2}\, dydx + \int_1^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\, dydx.$$

10. (10 points) Use Lagrange multiplier to find the extreme value of

$$f(x,y,z) = e^{xyz}$$

subject to the constraint

$$x^3 - y^3 + z^3 = 24.$$

Also, indicate the value(s) you obtain is (are) maximum or minimum.