

臺灣綜合大學系統 111 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D04
		科目碼	D0491

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 1 頁

1. (20 points) Are the following four functions even or odd or neither even nor odd? Show the details of your work.

$$e^x, e^{x^2}, \tan x, \cos x, \sinh x$$

2. (10 points) $f = x + y - z, g = xyz$. Please calculate $\text{div}(\text{grad}(fg))$?

3. (15 points) Please find the eigenbasis for the following matrix and diagonalize the matrix.

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

4. (10 points) Using the Laplace transform to solve the following ODE,

$$y_1' - 2y_1 + 3y_2 = 0, y_2' - y_1 + 2y_2 = 0 \quad y_1(0) = 1, y_2(0) = 0$$

5. (15 points) Solve the following ODE. (hint: Frobenius method)

$$x(x-1)\ddot{y} + (3x-1)\dot{y} + y = 0$$

6. Solve the following ODEs.

(a) (10 points) $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, y(0) = -3$

(b) (10 points) $y^{iv} - 5\ddot{y} + 4y = 0$

7. (10 points) Determine the radius of convergence.

$$\sum_{m=0}^{\infty} \frac{x^{2m+1}}{(2m+1)!}$$