

臺灣綜合大學系統 112 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定所有考科均「不可」使用計算機。

本科試題共計 1 頁

1. (10%) Evaluate the limit.

$$\lim_{x \rightarrow 4} \frac{4-x}{2-\sqrt{x}}$$

2. (10%) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(1, 0)$.

3. (10%) If $f(x) = \int_2^{e^x} \sqrt{1+t^2} dt$, find $(f^{-1})'(0)$.

4. (10%) Find the length of the polar curve $r = \sin^3(\theta/3)$, $0 \leq \theta \leq \pi$.

5. (10%) Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{\sqrt{n}} x^n$.

6. (10%) Find the unit tangent vector and the unit normal vector for the curve $\mathbf{r}(t) = \cos 3t\mathbf{i} + \sin 3t\mathbf{j} + 4t\mathbf{k}$.

7. (10%) Find equations of the tangent plane and the normal line to the surface $x+2y+3z = \sin(xyz)$ at the point $(2, -1, 0)$.

8. (10%) Evaluate the integral $\int_0^{1/2} \int_{\sqrt{3}y}^{\sqrt{1-y^2}} x^2 y dx dy$.

9. (10%) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle ye^{xy}, xe^{xy} \rangle$ and C is the unit circle $x^2 + y^2 = 1$.

10. (10%) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (z^2 x + e^y)\mathbf{i} + (x^2 y + \cosh z)\mathbf{j} + (y^2 z + x)\mathbf{k}$ and S is the top half of sphere $x^2 + y^2 + z^2 = 1$.