科目名稱	微積分 B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計　1　頁

Show all your work and justify all your answers.

1. (a) (5 points) Evaluate $\lim\limits_{x \to 2} \dfrac{x^2 + 5}{x^3 - 2x}$.

   (b) (5 points) Evaluate $\displaystyle\int_{-1}^{1} 6x^5 + 3x^2 \, dx$.

2. (10 points) Evaluate $\displaystyle\int_{1}^{2} \dfrac{1}{x^2} \cos\left(\dfrac{\pi}{x}\right) dx$.

3. (10 points) Evaluate $\lim\limits_{x \to -\infty} \dfrac{\ln(8^x + 9^x)}{3x}$.

4. (10 points) Find an equation for the tangent line to the curve $x^2 + 4xy + y^3 + 5 = 0$ at the point $(2, -1)$.

5. (10 points) Let $f(x) = \dfrac{x}{(x-1)^2}$. Find the intervals on which f is increasing and the intervals on which f is decreasing.

6. (10 points) Evaluate $\displaystyle\int_{0}^{1} \dfrac{1}{2x + 10\sqrt{x} + 12} \, dx$.

7. (10 points) Evaluate $\displaystyle\int_{0}^{\pi/2} \int_{y}^{2y} \dfrac{\sin x}{x} \, dx \, dy + \int_{\pi/2}^{\pi} \int_{y}^{\pi} \dfrac{\sin x}{x} \, dx \, dy$.

8. (10 points) Use the method of Lagrange multipliers to find the minimum value of the function
$$f(x, y, z) = x - y + z$$
subject to the constraint $2x^2 + 4y^2 + 4z^2 = 9$.

9. (10 points) Let $f(x, y, z) = x^2 z e^y + xz^2$. In what direction does f increase most rapidly at the point $(1, \ln 2, 2)$? What is the maximum rate of increase?

10. (10 points) For $|x| < \dfrac{3}{2}$, the function $f(x) = \dfrac{2x^5}{16x^4 + 81}$ can be expressed as a power series $\displaystyle\sum_{n=0}^{\infty} a_n x^n$, where a_n's are constants. Find the value of a_{25}.