科目名稱	線性代數	類組代碼	A07.C11
		科目碼	A0702

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計　1　頁

1. [10%] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 3 \\ 3 & 3 & 5 & 5 \end{bmatrix}.$$

Find a basis of the kernel of A.

2. [15%] Let $\alpha = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ be a basis of a vector space V. Suppose $\mathbf{y}_1 = 2\mathbf{x}_1 + 3\mathbf{x}_2$. Show that $\beta = \{\mathbf{y}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is also a basis of V.

3. [15%] Let I_n be the identity matrix of order n and J_n be the $n \times n$ all-ones matrix. Find the determinant of

$$A_n = \begin{bmatrix} I_n & -I_n \\ -I_n & J_n \end{bmatrix}.$$

4. [20%] Let

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \text{ and } L = \begin{bmatrix} 2 & 0 & -1 & -1 \\ 0 & 2 & -1 & -1 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}.$$

Let $\beta = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $W = \text{span}(\beta)$. Define the function $f : W \to W$ by $f(\mathbf{w}) = L\mathbf{w}$.

(a) [12%] Find the matrix representation $[f]_\beta^\beta$ of f with respect to β.

(b) [8%] Find the spectrum of f.

5. [20%] Let A be a square matrix. Suppose $\mathbf{v}_1, \ldots, \mathbf{v}_q$ are eigenvectors of A with respect to distinct eigenvalues $\lambda_1, \ldots, \lambda_q$. Show that $\{\mathbf{v}_1, \ldots, \mathbf{v}_q\}$ is linearly independent.

6. [20%] Let $C[0, 1]$ be the vector space of continuous real-valued functions on the interval $[0, 1]$ with the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx.$$

Find the orthogonal projection of e^x onto the space $\text{span}\{1, x\}$.