

臺灣綜合大學系統112學年度學士班轉學生聯合招生考試試題

科目名稱	微積分C	類組代碼	共同考科
		科目碼	E0013

※本項考試依簡章規定所有考科均「不可」使用計算機。

本科試題共計 1 頁

1. (10 points) Evaluate the limit

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}.$$

2. (10 points) Evaluate

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx.$$

3. (10 points) Evaluate

$$\int_0^1 \frac{1}{x^2 + 2} \, dx.$$

4. (10 points) Find the arc length of the cycloid

$$C: \begin{cases} x = r(\theta - \sin \theta) \\ y = r(1 - \cos \theta) \end{cases}, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

5. (10 points) Find the 5th derivative $f^{(5)}(0)$ of the function $f(x) = \ln(1+x) \cdot \tan^{-1} x$.

6. (10 points) Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 0$ and the cylinder $x^2 + y^2 = 29$.

7. (10 points) Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 9$.

8. (10 points) Evaluate

$$\iiint_B e^{(x^2+y^2+z^2)^{\frac{3}{2}}} \, dV,$$

where $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$ is the unit ball.

9. (10 points) Evaluate

$$\oint_C (y^2 + \sin x) \, dx + (3xy - e^y) \, dy$$

where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

10. (10 points) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = xy \mathbf{i} + (y^2 + e^{xz}) \mathbf{j} + \cos(xy) \mathbf{k}$$

and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0, y = 0, y + z = 2$.