

臺灣綜合大學系統 112 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D36
		科目碼	D3691

※本項考試依簡章規定所有考科均「不可」使用計算機。

本科試題共計 1 頁

- (20%) Obtain the solution of the following ODE
 - $y''' + y' = \sin x$ (10%)
 - $y'' - 4y = -7e^{2x} + x, y(0) = 1, y'(0) = 3$ (10%)
- (15%) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $P = A(A^T A)^{-1} A^T = \frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$, What is P^3 ?
- (15%) The inverse of a 2 by 2 matrix seems to have determinant = 1:

$$\det(A^{-1}) = \det\left(\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right) = \frac{ad-bc}{ad-bc} = 1.$$
 What is wrong with this calculation? What is the correct $\det(A^{-1})$?
- (15%)
 - Find the Fourier Sine series of the function $f(x) = 1, 0 < x < \pi$ (8%)
 - Using the results of (a), show that $\sin 1 + \frac{1}{3} \sin 3 + \frac{1}{5} \sin 5 + \dots = \frac{1}{4}\pi$. (7%)
- (20%) Solve the heat problem $u_t = k u_{xx}$ in $0 < x < L$, with the boundary conditions $u_x(0, t) = u_x(L, t) = 0$ and initial condition $u(x, 0) = f(x)$. What is $u(x, t)$ in the bar after a long time (theoretically, as $t \rightarrow \infty$)?
- (15%) Let $f(x, y) = \ln(x^2 + y^2 + 1) + e^{2xy}$
 - Find the gradient of f at the point $(0, -2)$ (5%)
 - Find the directional derivative of f at the point $(0, -2)$ in the direction of the vector $\mathbf{v} = 7\mathbf{i} - 24\mathbf{j}$ (5%)
 - Find the minimum value of the directional derivative at the point $(0, -2)$. (5%)