臺灣綜合大學系統 112 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D37
		科目碼	D3792

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 3 頁

The set $B=\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$,where $\mathbf{u}_1=\langle 1,1,1\rangle$, $\mathbf{u}_2=\langle 9,-1,1\rangle$, $\mathbf{u}_3=\langle -1,4,-2\rangle$ is a basis for R^3.
Transform B into an orthonormal basis $B''=\{\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\}$.

1. (6%) Where $\mathbf{w}_1$ is

(A) $\mathbf{w}_1=\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\rangle$ (B) $\mathbf{w}_1=\left\langle \frac{3}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}}\right\rangle$ (C) $\mathbf{w}_1=\left\langle \frac{1}{\sqrt{42}},\frac{4}{\sqrt{42}},\frac{-5}{\sqrt{42}}\right\rangle$

(D) $\mathbf{w}_1=\left\langle \frac{6}{\sqrt{35}},\frac{2}{\sqrt{35}},\frac{-9}{\sqrt{35}}\right\rangle$ (E) $\mathbf{w}_1=\left\langle \frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right\rangle$

2. (6%) Where $\mathbf{w}_2$ is

(A) $\mathbf{w}_2=\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\rangle$ (B) $\mathbf{w}_2=\left\langle \frac{3}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}}\right\rangle$ (C) $\mathbf{w}_2=\left\langle \frac{1}{\sqrt{42}},\frac{4}{\sqrt{42}},\frac{-5}{\sqrt{42}}\right\rangle$

(D) $\mathbf{w}_2=\left\langle \frac{6}{\sqrt{35}},\frac{2}{\sqrt{35}},\frac{-9}{\sqrt{35}}\right\rangle$ (E) $\mathbf{w}_2=\left\langle \frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right\rangle$

3. (6%) Where $\mathbf{w}_3$ is

(A) $\mathbf{w}_3=\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\rangle$ (B) $\mathbf{w}_3=\left\langle \frac{3}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}}\right\rangle$ (C) $\mathbf{w}_3=\left\langle \frac{1}{\sqrt{42}},\frac{4}{\sqrt{42}},\frac{-5}{\sqrt{42}}\right\rangle$

(D) $\mathbf{w}_3=\left\langle \frac{6}{\sqrt{35}},\frac{2}{\sqrt{35}},\frac{-9}{\sqrt{35}}\right\rangle$ (E) $\mathbf{w}_3=\left\langle \frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right\rangle$

Find an LU-factorization of $\mathbf{A}=\begin{bmatrix}4&9&7\\1&3&4\\2&5&3\end{bmatrix}$

4. (6%) Lower matrix $\mathbf{L}$

(A) $\mathbf{L}=\begin{bmatrix}4&0&0\\9&3&0\\7&4&3\end{bmatrix}$ (B) $\mathbf{L}=\begin{bmatrix}1&0&0\\\frac{1}{4}&1&0\\\frac{1}{2}&\frac{2}{3}&1\end{bmatrix}$ (C) $\mathbf{L}=\begin{bmatrix}1&0&0\\1&\frac{1}{4}&0\\\frac{2}{3}&-\frac{1}{2}&1\end{bmatrix}$ (D) $\mathbf{L}=\begin{bmatrix}\frac{1}{4}&0&0\\1&\frac{1}{2}&0\\1&1&\frac{2}{3}\end{bmatrix}$

(E) $\mathbf{L}=\begin{bmatrix}1 & 0 & 0\\ \frac{1}{4} & 1 & 0\\ \frac{1}{2} & \frac{2}{3} & 1\end{bmatrix}$

5. (6%) Upper matrix $\mathbf{U}$

(A) $\mathbf{U}=\begin{bmatrix}1 & \frac{1}{4} & 9\\ 0 & 1 & \frac{1}{4}\\ 0 & 0 & \frac{1}{4}\end{bmatrix}$ (B) $\mathbf{U}=\begin{bmatrix}\frac{1}{4} & 1 & -9\\ 0 & \frac{1}{4} & 1\\ 0 & 0 & \frac{1}{4}\end{bmatrix}$ (C) $\mathbf{U}=\begin{bmatrix}7 & \frac{3}{4} & 2\\ 0 & -\frac{9}{4} & 9\\ 0 & 0 & 4\end{bmatrix}$ (D) $\mathbf{U}=\begin{bmatrix}2 & 9 & 7\\ 0 & -\frac{9}{4} & \frac{3}{4}\\ 0 & 0 & 4\end{bmatrix}$

(E) $\mathbf{U}=\begin{bmatrix}4 & 9 & 7\\ 0 & \frac{3}{4} & \frac{9}{4}\\ 0 & 0 & -2\end{bmatrix}$

6. (6%) Diagonalize $\mathbf{A}=\begin{bmatrix}6 & -1\\ 2 & 3\end{bmatrix}$ if possible.

(A) $\mathbf{D}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (B) $\mathbf{D}=\begin{bmatrix}4 & 0\\ 0 & 5\end{bmatrix}$ (C) $\mathbf{D}=\begin{bmatrix}5 & 0\\ 0 & 4\end{bmatrix}$ (D) $\mathbf{D}=\begin{bmatrix}5 & 0\\ 0 & -4\end{bmatrix}$ (E) do not exist

7. (6%) If $\mathbf{A}^{-1}=\begin{bmatrix}4 & 9\\ 1 & 3\end{bmatrix}$ what is $\mathbf{A}$?

(A) $\mathbf{A}=\begin{bmatrix}\frac{1}{4} & \frac{1}{9}\\ 1 & \frac{1}{3}\end{bmatrix}$ (B) $\mathbf{A}=\begin{bmatrix}1 & 3\\ \frac{1}{3} & \frac{4}{3}\end{bmatrix}$ (C) $\mathbf{A}=\begin{bmatrix}1 & -3\\ -\frac{1}{3} & \frac{4}{3}\end{bmatrix}$ (D) $\mathbf{A}=\begin{bmatrix}-1 & 3\\ \frac{1}{3} & -\frac{4}{3}\end{bmatrix}$

(E) $\mathbf{A}=\begin{bmatrix}-\frac{1}{4} & \frac{1}{9}\\ 1 & -\frac{1}{3}\end{bmatrix}$

8. (6%) Find the eigenvalues of the given matrix $\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 1 & 3 & 3\end{bmatrix}$

(A) $\lambda=5, \lambda=-1, \lambda=1$ (B) $\lambda=1, \lambda=-5, \lambda=2$ (C) $\lambda=1, \lambda=-\sqrt{5}, \lambda=\sqrt{5}$ (D) $\lambda=1, \lambda=-\frac{1}{5}, \lambda=\frac{1}{5}$

(E) $\lambda=-5, \lambda=-1, \lambda=2$

背面有題，請繼續作答。

9. (10%) Show that the given integral $\int_{(1,2)}^{(3,4)} (2x+3)dx + 6y^2 dy$ is independent or dependent of the path. Evaluate.

(A) No exist, (B) independent and 47, (C) dependent and 47, (D) independent and 126, (E) dependent and 126.

10. (10%) Find the rank of the given matrix $\begin{cases} x_1 - 2x_2 + x_3 = 2 \\ 3x_1 - x_2 + 2x_3 = 5 \\ 2x_1 + x_2 + x_3 = 1 \end{cases}$ and determinate how many solutions this system has.

(A) 3 and unique solution, (B) 2 and unique solution, (C) 3 and infinity solution, (D) 2 and infinity solution, (E) no solution.

11. (10%) Please solve the initial value problem (IVP).

$y'' - 8y' + 16y = e^{4x}$, $y(0) = 0$, $y'(0) = 0$

(A) $y = \frac{1}{4}x^4 e^{4x}$ (B) $y = \frac{1}{8}x^2 e^{4x}$ (C) $y = \frac{1}{4}x^2 e^{4x}$ (D) $y = \frac{1}{2}x^2 e^{4x}$ (E) $y = \frac{1}{2}x^4 e^{4x}$

The position of a moving particle is given by $\mathbf{r}(t) = 6\cos t\mathbf{i} + 6\sin t\mathbf{j} - 2t\mathbf{k}$.

12. (6%) Find the vectors $\mathbf{T}$

(A) $\mathbf{T} = \left\langle \frac{-2}{\sqrt{10}}, \frac{-6}{\sqrt{10}}\sin t, \frac{6}{\sqrt{10}}\cos t \right\rangle$ (B) $\mathbf{T} = \left\langle \frac{-2}{\sqrt{10}}, \frac{-6}{\sqrt{10}}\sin t, \frac{6}{\sqrt{10}}\cos t \right\rangle$

(C) $\mathbf{T} = \left\langle \frac{-6}{\sqrt{10}}\sin t, \frac{-2}{\sqrt{10}}, \frac{6}{\sqrt{10}}\cos t \right\rangle$ (D) $\mathbf{T} = \left\langle \frac{-3}{\sqrt{10}}\sin t, \frac{3}{\sqrt{10}}\cos t, \frac{-1}{\sqrt{10}} \right\rangle$

(E) $\mathbf{T} = \left\langle \frac{-6}{\sqrt{10}}\sin t, \frac{6}{\sqrt{10}}\cos t, \frac{-2}{\sqrt{10}} \right\rangle$

13. (6%) Find the vectors $\mathbf{N}$

(A) $\mathbf{N} = \langle 0, \sin t, \cos t \rangle$ (B) $\mathbf{N} = \langle 1, \sin t, \cos t \rangle$ (C) $\mathbf{N} = \langle \sin t, 0, \cos t \rangle$

(D) $\mathbf{N} = \langle \sin t, -\cos t, 0 \rangle$ (E) $\mathbf{N} = \langle -\sin t, \cos t, 0 \rangle$

14. (5%) Find the curl of the vector $\mathbf{F}(x,y,z) = x^3 y\mathbf{i} + (y-z)\mathbf{j} + yz^2\mathbf{k}$.

(A) $\langle -x^3, 0, z^2+1 \rangle$ (B) $\langle z^2+1, 0, -x^3 \rangle$ (C) $(-x^3, 0, z^2+1)$ (D) $(z^2+1, 0, -x^3)$

(E) $(z^2+1, -x^3, 0)$

15. (5%) Find the divergence of the vector $\mathbf{F}(x,y,z) = x^3 y\mathbf{i} + (y-z)\mathbf{j} + yz^2\mathbf{k}$.

(A) $\langle -x^3, 0, z^2+1 \rangle$ (B) $\langle z^2+1, 0, -x^3 \rangle$ (C) $-3x^2y - 1 - 2yz$ (D) $3x^2y + 1 + 2yz$

(E) $(z^2+1, -x^3, 0)$