Show all your work and justify all your answers.

1. (a) (5 points) Evaluate $\displaystyle\lim_{x\to 2}\frac{|6x-17|-|6x-7|}{3x-6}$.

   (b) (5 points) Evaluate $\displaystyle\lim_{x\to 0}\left[\cos(2x)\right]^{x^{-2}}$.

2. (10 points) Let C be the curve defined by the parametric equations $x = t^3+1$, $y = t^4+t$. Find the slope of the tangent line to C at the point $(0,0)$.

3. (10 points) If $y^5 + 5xy + 1 = 0$, find the value of $\dfrac{d^2y}{dx^2}$ at the point $(0,-1)$.

4. (10 points) Find the absolute maximum value of the function $f(x) = x^8 e^{1-x^2}$.

5. (10 points) Evaluate $\displaystyle\int_0^3 \sqrt{6x - x^2}\, dx$.

6. (10 points) Let R be the region bounded by the curve $y = x^2 - x^7$ and the x-axis. Find the volume of the solid obtained by rotating R about the line $x = 3$.

7. (10 points) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{n(x-3)^n}{2^n(n^2+1)}$.

8. (10 points) Let $f(x,y,z) = x^2 + y^2 - 4xy + z + 1$. Find all possible numbers $a, b, c \in \mathbb{R}$ such that the direction in which f increases most rapidly at the point (a,b,c) is in the direction of $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

9. (10 points) Evaluate $\displaystyle\iint_D \cos(x+y)\, dA$, where $D = \left\{(x,y) \in \mathbb{R}^2 \ \middle|\ |x| + |y| \le \dfrac{\pi}{6}\right\}$.

10. (10 points) Let $\mathbf{F}$ be the vector field defined by

$$\mathbf{F}(x,y,z) = \left(9x^2 z\right)\mathbf{i} + \left(8\sin(x^3) + e^{2z}\right)\mathbf{j} + \left(x^5 y \ln(x^2+1)\right)\mathbf{k},$$

and let S be the surface of the solid bounded by the planes $x + 3z = 6$, $y = 3$, $x = 0$, $y = 0$, and $z = 0$. Suppose S is given with positive (outward) orientation. Evaluate the flux of $\mathbf{F}$ across S.