臺灣綜合大學系統 113 學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D37
		科目碼	D3792

※本項考試依簡章規定所有考科均「不可」使用計算機。　本科試題共計 3 頁

1. (6%) Evaluate $\oint_c \frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dx$, where C is the boundary of the shaded region R shown in Fig.(a)

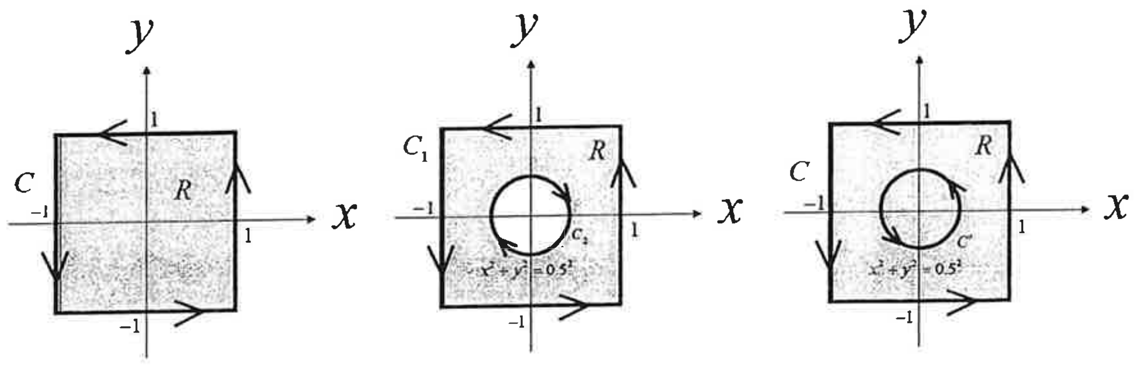
(A) 2π (B) 0 (C) π (D) $\frac{\pi}{4}$ (E) do not exist.

2. (6%) Evaluate $\oint_c \frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dx$, where $C=C_1\cup C_2$ is the boundary of the shaded region R shown in Fig.(b)

(A) 2π (B) 0 (C) π (D) $\frac{\pi}{4}$ (E) do not exist.

3. (6%) Evaluate $\oint_c \frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dx$, where $C=C\cup C'$ is the boundary of the shaded region R shown in Fig.(c)

(A) 2π (B) 0 (C) π (D) $\frac{\pi}{4}$ (E) do not exist.



(a) (b) (c)

4. (6%) Find the curl of the vector $\mathbf{F}(x,y,z)=5yz\mathbf{i}+x^2z\mathbf{j}+3x^3\mathbf{k}$.

(A) $-xy-5z$, (B) $\langle 5y-9x^2, 2xz-5z, -x^2\rangle$, (C) $\mathbf{0}$, (D) 0 ,(E) $\langle -x^2, 5y-9x^2, 2xz-5z\rangle$.

5. (6%) Find the divergence of the vector $\mathbf{F}(x,y,z)=ye^{5xy}\mathbf{i}+x^2\sin yz\mathbf{j}+\cos xz^3\mathbf{k}$.

(A) $\langle -x^2\cos yz, z^3\sin xz^3, 2x\sin yz-e^{5xy}-5xe^{5xy}\rangle$

(B) $\langle z^3\sin xz^3, -x^2\cos yz, 2x\sin yz-e^{5xy}-5xe^{5xy}\rangle$

(C) $5y^2e^{5xy}+zx^2\cos yz-3xz^2\sin xz^3$ (D) $5y^2\cos yz+zx^2e^{5xy}-3xz^2\sin xz^3$ (E) 0

6. (6%) Please solve the initial value problem (IVP).

$y = c_1 e^{-x} + c_2 e^{x},\ y(0) = 0,\ y'(0) = 1$

(A) $y = \frac{1}{2}e^{-x} + \frac{1}{2}e^{x}$, (B) $y = -\frac{1}{2}e^{-x} + \frac{1}{2}e^{x}$, (C) $y = -\frac{1}{2}e^{-x} - \frac{1}{2}e^{x}$, (D) $y = \frac{1}{2}e^{-x} - \frac{1}{2}e^{x}$, (E) $y = \frac{1}{2}e^{-x}$.

7. (8%) Evaluate $\mathcal{L}\{f(t)\}, f(t) = \begin{cases} -1 & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$

(A) $\frac{2}{s}e^{-s} - \frac{2}{s}$, (B) $\frac{1}{s}e^{-s} - \frac{1}{s}$, (C) $\frac{2}{s}e^{-s} - \frac{1}{2s}$, (D) $\frac{1}{s}e^{-s} - \frac{2}{s}$, (E) $\frac{2}{s}e^{-s} - \frac{1}{s}$.

8. (8%) Find the rank of the given matrix $\begin{cases} x_1 - x_2 + 3x_3 = -1 \\ x_1 - 3x_2 + 4x_3 = 5 \\ x_1 - x_2 + 6x_3 = 2 \end{cases}$ and determinate how many solutions this system has.

(A) 2 and infinity solution, (B) 2 and unique solution, (C) 3 and infinity solution, (D) 3 and unique solution, (E) no solution.

9. (6%) Determine which of the indicated column vectors are eigenvectors of the given matrix

$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$; $\mathbf{K}_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$, $\mathbf{K}_2 = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{K}_3 = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

(A) All eigenvectors, (B) none of them, (C) $\mathbf{K}_1$, (D) $\mathbf{K}_2$, (E) $\mathbf{K}_3$.

10. (6%) Diagonalize $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ if possible.

(A) $\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$, (B) $\mathbf{D} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$, (C) $\mathbf{D} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, (D) $\mathbf{D} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$, (E) do not exist.

11. (6%) Find the orthogonal matrix $\mathbf{P}$ of $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ if possible.

(A) $\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, (B) $\mathbf{P} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, (C) $\mathbf{P} = \begin{bmatrix} -3 & 1 \\ 1 & 1 \end{bmatrix}$, (D) $\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, (E) do not exist.

12. (8%) If $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$ what is $\mathbf{A}$?

(A) $\mathbf{A} = \begin{bmatrix} \frac{5}{6} & \frac{-2}{3} \\ \frac{-1}{6} & \frac{1}{3} \end{bmatrix}$, (B) $\mathbf{A} = \begin{bmatrix} \frac{-2}{3} & \frac{5}{6} \\ \frac{1}{3} & \frac{-1}{6} \end{bmatrix}$, (C) $\mathbf{A} = \begin{bmatrix} \frac{-1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{-2}{3} \end{bmatrix}$, (D), $\mathbf{A} = \begin{bmatrix} \frac{-2}{3} & \frac{1}{3} \\ \frac{5}{6} & \frac{-1}{6} \end{bmatrix}$

(E) $\mathbf{A} = \begin{bmatrix} \frac{-5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{-1}{3} \end{bmatrix}$.

13. (8%) The given integral $\int_{(1,0)}^{(3,2)}(x+2y)dx+(2x-y)dy$ is independent or dependent of the path. Evaluate.

(A) No exist, (B) independent and 14, (C) dependent and 14, (D) independent and 126,

(E) dependent and 126.

$\iint_R (x^2+y^2)\sin xy dA$, were R is the region in the first quadrant bounded by the graph of $x^2-y^2=1$, $x^2-y^2=9$, $xy=2$, $xy=-2$; $u=x^2-y^2$, $v=xy$

14. (6%) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$

(A) $\begin{bmatrix} 2x & -2y \\ y & x \end{bmatrix}$, (B) $\begin{bmatrix} -2x & 2y \\ x & y \end{bmatrix}$, (C) $2(x^2+y^2)$, (D) $\frac{-1}{2(x^2+y^2)}$ (E) 1.

15. (8%) Evaluate the given integral

(A)6 , (B) 4 , (C) 2, (D) 0, (E) −4.